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A NEW CLASS OF INTERVAL VALUED VAGUE SPACE

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ABSTRACT:

In this paper we have introduced several characterizations of interval valued vague resolvable spaces and irresolvable spaces. Various examples are given to explain the concepts introduced.

Keywords: Interval valued vague resolvable, Interval valued vague irresolvable, Interval valued vague open hereditarily irresolvable.

1. INTRODUCTION:

The notion of vague set was originated by Gau and Buehrer[3] in the year 1994. Collaborating the idea of interval valued fuzzy sets and vague sets, Zhifeng et al[14], in 2001 initiated the concept of interval vague sets (IVSs). Many authors such as Gau and Buehrer[3], Li and Rao[6], Liu. P. D[9] and Liu. P. D and Guan [7,8] have extended the notion of the required operations of vague sets and interval valued vague sets. Interval valued vague sets have been applied in different area and it is one of the higher order fuzzy sets. The concept of truth membership function and false membership function in interval valued vague sets relates the world more practically. In classical topology , the theory of resolvability and irresolvability in topological spaces was established by E. Hewitt [4] and A.G. El'kin [2] introduced open hereditarily irresolvable spaces. Thangaraj.G and Balasubramanian. G[13] introduced the concept of fuzzy resolvable space and fuzzy irresolvable space. In this paper we introduce the concept of interval valued vague resolvable space and irresolvable space. Various examples are given to demonstrate the concepts introduced in this paper.

2. Preliminaries:

Definition 2.1: [5] Let [I] be the set of all closed subintervals of the interval [0,1] and $\mu = [\mu_L, \mu_U] \in [I]$, where μ_L and μ_U are the lower extreme and the upper extreme, respectively. For a set X, an IVFS A is given by equation $A = \{< x, \mu_A(x) > / x \in X\}$ where the function $\mu_A : X \to [I]$ defines the degree of membership of an element x to A, and $\mu_A(x) = [\mu_{AL}(x), \mu_{AU}(x)]$ is called an interval valued fuzzy number.

Definition 2.2: [3] A vague set A in the universe of discourse U is characterized by two membership functions given by:

(i) A true membership function $t_A : U \to [0,1]$ and

(ii) A false membership function $f_A : U \to [0,1]$

where $t_A(x)$ is a lower bound on the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence for x", and $t_A(x) + f_A(x) \le 1$. Thus the grade of membership of u in the vague set A is bounded by a subinterval $[t_A(x), 1 - f_A(x)]$ of [0,1]. This indicates that if the actual grade of membership of x is μ(x), then, $t_A(x) \le \mu(x) \le 1 - f_A(x)$.The vague set A is written as $A = \{\langle x, [t_A(x), 1 - f_A(x)] \rangle / u \in U\}$ where the interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A, denoted by $V_A(x)$.

Definition 2.3:[12] An interval valued vague sets $\tilde{A}^V$ over a universe of discourse X is defined as an object of the form $\tilde{A}^V = \{< x_i, [T_{\tilde{A}^V}(x_i), F_{\tilde{A}^V}(x_i)] >, x_i \in X\}$ where $T_{\tilde{A}^V} : X \to D([0,1])$ and $F_{\tilde{A}^V} : X \to D([0,1])$ are called " truth membership function" and "false membership function" respectively and where D[0,1] is the set of all intervals within [0,1], or in other word an interval valued vague set can be represented by $\widetilde{A}^V = <[(x_i),[\mu_1,\mu_2],[v_1,v_2]]>, x_i \in X$ where $0 \le \mu_1 \le \mu_2 \le 1$ *and* $0 \le v_1 \le v_2 \le 1$. For each interval valued vague set $\widetilde{A}^V$, $\pi_{1\widetilde{A}^V}(x_i) = 1 - \mu_{1\widetilde{A}^V}(x_i) - v_{1\widetilde{A}^V}(x_i)$ are called degree of hesitancy of x_i in $\widetilde{A}^V$ respectively.

Definition 2.4:[10] An interval valued vague topology (IVT in short) on X is a family τ of interval valued vague sets(IVS) in X satisfying the following axioms.
(i) $0, 1 \in \tau$
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$
(iii) $\bigcup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.
In this case the pair (X, τ) is called an interval valued vague topological space (IVTS in short) and any IVS in τ is known as a Interval valued vague open set(IVOS in short) in X.
The complement A^c of a IVOS A in a IVTS (X, τ) is called an interval valued vague closed set (IVCS in short) in X.

Definition 2.5:[10] Let $A = \{\langle x,[t_A^L(x),t_A^U(x)],[1-f_A^L(x),1-f_A^U(x)]\rangle\}$ and

$B = \{\langle x,[t_B^L(x),t_B^U(x)],[1-f_B^L(x),1-f_B^U(x)]\rangle\}$ be two interval valued vague sets then their union, intersection and complement are defined as follows:

(i) $A \bigcup B = \{< x,[t_{A\cup B}^L(x),t_{A\cup B}^U(x)],[1-f_{A\cup B}^L(x),1-f_{A\cup B}^U(x)] > / x \in X\}$ where

$t_{A\cup B}^L(x) = \max\{t_A^L(x),t_B^L(x)\},\ t_{A\cup B}^U(x) = \max\{t_A^U(x),t_B^U(x)\}$ *and*
$1-f_{A\cup B}^L(x) = \max\{1-f_A^L(x),1-f_B^L(x)\},\ 1-f_{A\cup B}^U(x) = \max\{1-f_A^U(x),1-f_B^U(x)\}$

(ii) $A \bigcap B = \{< x,[t_{A\cap B}^L(x),t_{A\cap B}^U(x)],[1-f_{A\cap B}^L(x),1-f_{A\cap B}^U(x)] > / x \in X\}$ where

$t_{A\cap B}^L(x) = \min\{t_A^L(x),t_B^L(x)\},\ t_{A\cap B}^U(x) = \min\{t_A^U(x),t_B^U(x)\}$ *and*
$1-f_{A\cap B}^L(x) = \min\{1-f_A^L(x),1-f_B^L(x)\},\ 1-f_{A\cap B}^U(x) = \min\{1-f_A^U(x),1-f_B^U(x)\}$

(iii) $\overline{A} = \{< x,[f_A^L(x),f_A^U(x)],[1-t_A^L(x),1-t_A^U(x)] > / x \in X\}$.

Definition 2.6:[10] Let (X, τ) be an interval valued vague topological space and $A = \{\langle x,[t_A^L,t_A^U],[1-f_A^L,1-f_A^U]\rangle\}$ be a IVS in X. Then the interval valued vague interior and an interval valued vague closure are defined by
$IV\operatorname{int}(A) = \bigcup$ {G/G is an IVOS in X and G $\subseteq$ A}
$IVcl(A) = \bigcap$ {K/K is an IVCS in X and A $\subseteq$ K}
Note that for any IVS A in (X, τ), we have $IVcl(A^c) = (IV\operatorname{int}(A))^c$ and $V\operatorname{int}(A^c) = (IVcl(A))^c$. and $IVcl(A)$ is an IVCS and $IV\operatorname{int}(A)$ is an IVOS in X. Further we have, if A is an IVCS in X thenIVcl(A)=A and if A is an IVOS in X then IVint(A)=A.

Definition 2.7:[10] An interval valued vague set A in an interval valued vague topological space (X, τ) is called an interval valued vague dense if there exists no interval valued vague closed set B in (X, τ) such that $A \subset B \subset 1$.

Definition 2.8:[10] An interval valued vague set A in an interval valued vague topological space (X, τ) is called an interval valued vague nowhere dense set if there exists no interval valued vague open set B in (X, τ) such that $B \subset IVcl(A)$. That is, $IV\operatorname{int}(IVcl(A)) = 0$.

Definition 2.9:[10] An interval valued vague topological space (X,τ) is called an interval valued vague first category set if $A=\bigcup_{i=1}^{\infty}(A_i)$, where A_i's are interval valued vague nowhere dense sets in (X,τ). Any other interval valued vague set in (X,τ) is said to be of interval valued vague second category.

Definition 2.10:[10] An interval valued vague set A in an interval valued vague topological space (X,τ) is called an interval valued vague G_δ-sets in (X,τ) if $A=\bigcap_{i=1}^{\infty}(A_i)$ where $A_i \in \tau$, for $i \in I$.

Definition 2.11:[10] An interval valued vague set A in an interval valued vague topological space (X,τ) is called an interval valued vague F_σ-sets in (X,τ) if $A=\bigcup_{i=1}^{\infty}(A_i)$ where $A_i^c \in \tau$, for $i \in I$.

Definition 2.12:[10] An interval valued vague topological space (X,τ) is called an interval valued vague Volterra space if $IVcl(\bigcap_{i=1}^{N} A_i)=1$, where A_i's are interval valued vague dense and interval valued vague G_δ-sets in (X,τ).

Definition 2.13:[11] An interval valued vague topological space (X,τ) is called an interval valued vague weakly Volterra space if $IVcl(\bigcap_{i=1}^{N} A_i)\neq 0$, where A_i's are interval valued vague dense and interval valued vague G_δ-set in (X,τ).

Definition 2.14:[10] Let (X,τ) be an interval valued vague topological space. Then (X,τ) is called an interval valued vague baire space if $IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i)=0$ where A_i's are interval valued vague nowhere dense sets in (X,τ).

Definition 2.15:[11] Let (X,τ) be an interval valued vague topological space. An interval valued vague set A in (X,τ) is called an interval valued vague σ- nowhere dense set if A is an interval valued vague F_σ set in (X,τ) such that $IV\operatorname{int}(A)=0$.

Definition 2.16:[11] Let (X,τ) be an interval valued vague topological spaces. Then (X,τ) is called an interval valued vague σ- Baire space if $IV\operatorname{int}(\bigcup_{i=1}^{\infty} A_i)=0$, where A_i's are interval valued vague σ- nowhere dense set (X,τ).

Definition 2.17:[11] An interval valued vague topological space (X,τ) is called an interval valued vague submaximal space if for each interval valued vague set A in (X,τ) such that $IVcl(A)=1$, then $A\in\tau$.

Definition 2.18:[11] Let (X,τ) be an interval valued vague topological space. An interval valued vague set A in (X,τ) is called interval valud vague σ- first category if $A=\bigcup_{i=1}^{\infty} A_i$ where A_i's are interval valued vague σ- nowhere dense set in (X,τ). Any other interval valued vague set in (X,τ) is said to be interval valued vague σ - second category in (X,τ).

Definition 2.19:[11] An interval valued vague topological space (X,τ) is an interval valued vague σ- first category space if $1=\bigcup_{i=1}^{\infty} A_i$, where A_i's are interval valued vague σ- nowhere dense set in (X,τ). (X,τ) is called an interval valued vague vague σ - second category space if it is not an interval valued vague σ - first category space.

3. Interval valued vague resolvable and irresolvable space:

Definition 3.1: Let (X, τ) be an interval valued vague topological space. (X, τ) is called an interval valued vague resolvable if there exists an interval valued vague dense set A in (X, τ) such that $IVcl(A^c)=1$. Otherwise, (X, τ) is called interval valued vague irresolvable.

Example 3.2: Let X={a,b}. Define an interval valued vague sets A, B and C as follows, $A=\{<x,[[0.3,0.4],[0.7,0.8]],[[0.4,0.5],[0.8,0.9]]>\}$, $B=\{<x,[[0.1,0.2],[0.5,0.6]],[[0.2,0.3],[0.4,0.5]]>\}$

$C=\{<x,[[0.2,0.3],[0.6,0.7]],[[0.3,0.5],[0.5,0.7]]>\}$

Clearly $\tau=\{0,1,A\}$ is an interval valued vague topology in (X, τ). Now $IV\operatorname{int}(B)=0$, $IV\operatorname{int}(C)=0$, $IV\operatorname{int}(B^c)=0$, $IV\operatorname{int}(C^c)=0$, $IVcl(B)=1$, $IVcl(C)=1$, $IVcl(B^c)=1$ and $IVcl(C^c)=1$. Since $IVcl(B)=1$ there exists an interval valued vague dense set B in (X, τ). Therefore the interval valued vague topological space (X, τ) is called an interval valued vague resolvable.

Example 3.3: Let X={a,b}. Define an interval valued vague sets A, B and C as follows, $A=\{<x,[[0.2,0.3],[0.5,0.6]],[[0.1,0.2],[0.6,0.7]]>\}$, $B=\{<x,[[0.3,0.4],[0.8,0.9]],[[0.2,0.3],[0.7,0.8]]>\}$

$C=\{<x,[[0.4,0.5],[0.7,0.8]],[[0.4,0.5],[0.8,0.9]]>\}$

Clearly $\tau=\{0,1,A\}$ is an interval valued vague topology in (X, τ). Now $IV\operatorname{int}(B)=A$, $IV\operatorname{int}(c)=A$, $IVcl(B)=1$, $IVcl(C)=1$. Therefore B and C are interval valued vague dense set in (X, τ). Then we have $IVcl(B^c)=A^c$ and $IVcl(C^c)=A^c$. Hence the interval valued vague topological space (X, τ) is called an interval valued vague irresolvable.

Theorem 3.4: Let (X, τ) be an interval valued vague topological space. (X, τ) is an interval valued vague resolvable space iff (X, τ) has a pair of interval valued vague dense set A_1 and A_2 such that $A_1 \subseteq A_2^C$.

Proof: Let (X, τ) be an interval valued vague topological space. (X, τ) is an interval valued vague resolvable space. Suppose that for all interval valued vague dense set A_i and A_j, we have $A_i \not\subset A_j^C$. Then $A_i \supset A_j^C$. Then $IVcl(A_i) \supset IVcl(A_j^C)$ which implies that $1 \supset IVcl(A_j^C)$ then $IVcl(A_j^C) \neq 1$. Also $A_j \supset A_i^C$ then $IVcl(A_j) \supset IVcl(A_i^C)$ which implies that $1 \supset IVcl(A_i^C)$. Then $IVcl(A_i^C) \neq 1$. Hence $IVcl(A_i)=1$, but $IVcl(A_i^C) \neq 1$ for all interval valued vague set A_i in (X, τ) which is a contradiction. Hence (X, τ) has a pair of interval valued vague dense set set A_1 and A_2 such that $A_1 \subseteq A_2^C$.

Conversely, suppose that the interval valued vague topological space (X, τ) has a pair of interval valued vague dense set set A_1 and A_2 such that $A_1 \subseteq A_2^C$. Suppose that (X, τ) is an interval valued vague irresolvable space. Then for all interval valued vague dense set A_1 and A_2 in (X, τ), we have $IVcl(A_1^C) \neq 1$ Then $IVcl(A_2^C) \neq 1$ implies that there exists an interval valued vague closed set B in (X, τ), such that $A_2^C \subset B \subset 1$. Then $A_1 \subseteq A_2^C \subset B \subset 1$ implies that. $A_1 \subset B \subset 1$ which is a contradiction. Hence (X, τ) is an interval valued vague resolvable space.

Theorem 3.5: If (X, τ) is an interval valued vague irresolvable space iff $IV\,\text{int}(A) \neq 0$ for all interval valued vague dense set A in (X, τ).

Proof: Suppose (X, τ) is an interval valued vague irresolvable space, for all interval valued vague dense set A in (X, τ), $IVcl(A^C) \neq 1$. Then $(IV\,\text{int}(A))^C \neq 1$, which implies $IV\,\text{int}(A) \neq 0$. Conversely, $IV\,\text{int}(A) \neq 0$, for all interval valued vague dense set A in (X, τ). Suppose that (X, τ) is an interval valued vague resolvable. Then there exists an interval valued vague dense set A in (X, τ), such that $IVcl(A^C) = 1$ implies that $(IV\,\text{int}(A))^C = 1$, implies $IV\,\text{int}(A) = 0$, which is a contradiction. Hence (X, τ) is an interval valued vague irresolvable space.

Definition 3.6: An interval valued vague topological space (X, τ) is called an interval valued vague almost GP space if $IV\,\text{int}(A) \neq 0$, for each non-zero interval valued vague dense and interval valued vague G_δ-set A in (X, τ). That is, (X, τ) is an interval valued vague almost GP- space if for each non-zero interval valued vague G_δ-set A in (X, τ) with $IVcl(A) = 1,\ IV\,\text{int}(A) \neq 0$.

Theorem 3.7: If the interval valued vague topological space (X, τ) is an interval valued vague irresolvable space, then (X, τ) is an interval valued vague almost GP- space.

Proof: Let A be an interval valued vague dense and interval valued vague G_δ-set in an interval valued vague irresolvable space (X, τ). Since (X, τ) is an interval valued vague irresolvable space, for the fuzzy dense set A in (X, τ), we have $IVcl(A^C) \neq 1$. But $IVcl(A^C) = (IV\,\text{int}(A))^C \neq 1$, implies that $IV\,\text{int}(A) \neq 0$ and hence (X, τ) is an interval valued vague almost GP- space.

Theorem 3.8: If the interval valued vague topological space (X, τ) is interval valued vague submaximal, then (X, τ) is an interval valued vague irresolvable.

Proof: Let (X, τ) be an interval valued vague submaximal space. Assume that (X, τ) is an interval valued vague resolvable space. Let A be an interval valued vague dense set in (X, τ). Then $IVcl(A^C) = 1$. Hence $(IV\,\text{int}(A))^C = 1$, which implies that $IV\,\text{int}(A) = 0$. Then $A \notin T$, which is a contradiction to interval valued vague submaximal space of (X, τ). Hence (X, τ) is an interval valued vague irresolvable space.

Theorem 3.9: If (X, τ) is an interval valued vague Baire space and interval valued vague irresolvable space, then $IVcl(\bigcup_{i=1}^{\infty} A_i) \neq 1$, where A_i's are interval valued vague nowhere dense sets in (X, τ).

Proof: Let A be an interval valued vague first category set in (X, τ). Then $A = \bigcup_{i=1}^{\infty} A_i$, where A_i's are interval valued vague dense sets in (X, τ). Since (X, τ) is an interval valued vague Baire space, $IV\,\text{int}(A) = 0$. Then $(IV\,\text{int}(A))^C = 1$, which implies that $IVcl(A^C) = 1$. Since (X, τ) is an interval valued vague irresolvable space, $IVcl((A^C)^C) \neq 1$. Hence $IVcl(A) \neq 1$ and therefore $IVcl(\bigcup_{i=1}^{\infty} A_i) \neq 1$, where A_i's are interval valued vague nowhere dense sets in (X, τ).

Definition 3.10: An interval valued vague topological space (X, τ) is said to be an interval valued vague open hereditarily irresolvable if $IV\,\text{int}(IVcl(A)) \neq 0$ then $IV\,\text{int}(A) \neq 0$ for any interval valued vague set A in (X, τ).

Example 3.11: Let X={a,b}. Define an interval valued vague sets A, B and C as follows,
$A = \{< x, [[0.1,0.2],[0.5,0.6]], [[0.1,0.2],[0.7,0.8]] >\}$ $B = \{< x, [[0.4,0.5],[0.7,0.8]], [[0.2,0.3],[0.8,0.9]] >\}$
$C = \{< x, [[0.3,0.4],[0.6,0.7]], [[0.2,0.3],[0.6,0.7]] >\}$ Clearly
$\tau = \{0,1, A, B\}$ is an interval valued vague topology in (X, τ). Now $IVcl(A) = A^C$, $IVcl(B) = 1$ and $IVcl(C) = A^C$. Also $IV\,\text{int}(IVcl(A) = IVit(A^C) = A \neq 0$ and $IV\,\text{int}(A) = A \neq 0$,

$IV\,\text{int}(IVcl(B)) = IV\,\text{int}(1) = 1 \neq 0$ and $IV\,\text{int}(B) = B \neq 0$, $IV\,\text{int}(IVcl(C)) = A \neq 0$ and $IV\,\text{int}(C) = A \neq 0$, $IV\,\text{int}(IVcl(A^c)) = A \neq 0$, $IV\,\text{int}(IVcl(B^c)) = A \neq 0$ and $IV\,\text{int}(IVcl(C^c)) = 1 \neq 0$. Hence if $IV\,\text{int}(IVcl(A)) \neq 0$ then $IV\,\text{int}(A) \neq 0$ for any non zero interval valued vague set A in (X, τ). Therefore (X, τ) is an interval valued vague open hereditarily irresolvable space.

Theorem 3.12: If (X, τ) is an interval valued vague open hereditarily irresolvable space, then any interval valued vague σ -nowhere dense set in (X, τ) is an interval valued vague nowhere dense set in (X, τ).

Proof: Let A be an interval valued vague σ -nowhere dense set in an interval valued vague open hereditarily irresolvable space in (X, τ). Then A is an interval valued vague F_σ -set in (X, τ) such that $IV\,\text{int}(A) = 0$. Since (X, τ) is an interval valued vague open hereditarily irresolvable space, $IV\,\text{int}(A) = 0$ implies that $IV\,\text{int}(IVcl(A)) = 0$. Hence A is an interval valued vague nowhere dense set in (X, τ).

Theorem 3.13: Let (X, τ) be an interval valued vague topological space. If (X, τ) is an interval valued vague open hereditarily irresolvable then (X, τ) is an interval valued vague irresolvable.

Proof: Let A be an interval valued vague dense set in (X, τ). Then $IVcl(A) = 1$, which implies that $IV\,\text{int}(IVcl(A)) = 1 \neq 0$. Since (X, τ) is an interval valued vague open hereditarily irresolvable, we have $IV\,\text{int}(A) \neq 0$. Therefore by Theorem 3.5, $IV\,\text{int}(A) \neq 0$ for all interval valued vague dense set in (X, τ), implies that (X, τ) is an interval valued vague irresolvable. The converse is not true is shown by the example.

Example 3.14: Let X={a,b}. Define an interval valued vague sets A, B and C as follows, $A = \{< x, [[0.2,0.3],[0.5,0.6]],[[0.3,0.4],[0.6,0.7]] >\}$ $B = \{< x, [[0.3,0.4],[0.6,0.7]],[[0.4,0.5],[0.7,0.8]] >\}$ $C = \{< x, [[0.3,0.4],[0.7,0.8]],[[0.4,0.5],[0.6,0.7]] >\}$ Clearly $\tau = \{0,1,A\}$ is an interval valued vague topology in (X, τ). Now C and 1 are interval valued vague dense set in (X, τ). Then $IV\,\text{int}(C) = A \neq 0$, $IV\,\text{int}(1) \neq 0$. Then we have $IVcl(C^c) = A^c$. Hence (X, τ) is an interval valued vague irresolvable. But $IV\,\text{int}(IVcl(C^C) = IV\,\text{int}(A^C) = A \neq 0$ and $IV\,\text{int}(C^C) = 0$. Thus (X, τ) is not an interval valued vague open hereditarily irresolvable space.

Theorem 3.15: Let (X, τ) be an interval valued vague open hereditarily irresolvable. Then $IV(\text{int}(A) \not\subset (IV\,\text{int}(B))^C$ for any two interval valued vague dense sets A and B in (X, τ).

Proof: Let A and B be any two interval valued vague dense sets in (X, τ). Then $IVcl(A) = 1$and $IVcl(B) = 1$ implies that $IV\,\text{int}(IVcl(A)) \neq 0$ and $IV\,\text{int}(IVcl(B)) \neq 0$. Since (X, τ) is an interval valued vague open hereditarily irresolvable, $IV\,\text{int}(A) \neq 0$ and $IV\,\text{int}(B) \neq 0$. Hence by Theorem 3.4, $A \not\subset B^C$. Therefore $IV\,\text{int}(A) \subset A \not\subset B^C \subset (IV\,\text{int}(B))^C$. Hence we have $IV(\text{int}(A) \not\subset (IV\,\text{int}(B))^C$ for any two interval valued vague dense sets A and B in (X, τ).

Theorem 3.16: Let (X, τ) be an interval valued vague topological space. If (X, τ) is an interval valued vague open hereditarily irresolvable then $IV\,\text{int}(A) = 0$ for any nonzero interval valued vague dense set A in (X, τ) implies that $IV\,\text{int}(IVcl(A)) = 0$.

Proof: Let A be an interval valued vague set in (X, τ), such that $IV\,\text{int}(A) = 0$. We claim that $IV\,\text{int}(IVcl(A)) = 0$.. Suppose that $IV\,\text{int}(IVcl(A)) = 0$..Since (X, τ) is an interval valued vague open hereditarily irresolvable, we have $IV\,\text{int}(A) \neq 0$, which is a contradiction to $IV\,\text{int}(A) = 0$. Hence $IV\,\text{int}(IVcl(A)) = 0$.

Theorem 3.17: If the interval valued vague topological space (X, τ) is an interval valued vague open hereditarily irresolvable and interval valued vague second category space, then (X, τ) is an interval valued vague weakly volterra space.

Proof: Assume that (X, τ) is not an interval valued vague weakly volterra space. Therefore, $IVcl(\bigcap_{i=1}^{N} A_i) = 0$, where A_i 's are interval valued vague dense and interval valued vague G_δ-set in (X, τ).Then $IVcl(\bigcap_{i=1}^{N} A_i) = 0 \Rightarrow IV\,\text{int}(\bigcup_{i=1}^{N}(A_i)^c) = 1 \Rightarrow (\bigcup_{i=1}^{N}(A_i)^c) = 1$. Now $(\bigcup_{i=1}^{N}(A_i)^c) \subseteq (\bigcup_{i=1}^{\infty}(A_i)^c) \Rightarrow \bigcup_{i=1}^{\infty}(A_i)^c) = 1$, since $IVcl(A_i) = 1$, we have $IV\,\text{int}(A_i^C) = 0$. Since (X, τ) is an interval valued vague open hereditarily irresolvable, $IV\,\text{int}(A_i^C) = 0$, which implies that $IV\,\text{int}(IVcl(A_i^C) = 0$. Therefore A_i^C 's are interval valued vague nowhere dense set. Therefore $\bigcup_{i=1}^{\infty}(A_i)^c) = 1$, where A_i 's are interval valued vague nowhere dense sets. This implies that (X, τ) is an interval valued vague first category space, which is a contradiction since (X, τ) is an interval valued vague second category space. Hence (X, τ) is an interval valued vague weakly volterra space.

Theorem 3.18: Let (X, τ) be an interval valued vague topological space. If (X, τ) is an interval valued vague open hereditarily irresolvable then $IVcl(A) = 1$ for any nonzero interval valued vague dense set A in (X, τ) implies that $IVcl(IV\,\text{int}(A)) = 1$.

Proof: Let A be an interval valued vague set in (X, τ), such that $IVcl(A) = 1$. Then we have $(IVcl(A))^C = 0$, which implies that $IV\,\text{int}(A^C) = 0$. Since (X, τ) is an interval valued vague open hereditarily irresolvable by Theorem 3.16, we have that $IV\,\text{int}(IVcl(A^C)) = 0$. Therefore $(IVcl(IV\,\text{int}(A)))^C = 0$ implies that $(IVcl(IV\,\text{int}(A))) = 1$.

Theorem 3.19: If $IVcl(\bigcap_{i=1}^{\infty} A_i) = 1$, where A_i's are interval valued vague dense set in an interval valued vague open hereditarily irresolvable space, then (X, τ) is an interval valued vague Baire space.

Proof: Now $IVcl(\bigcap_{i=1}^{\infty} A_i) = 1$, where $IVcl(A_i) = 1$, implies that $IV\,\text{int}(\bigcup_{i=1}^{\infty}(A_i^C)) = 0$, where $IV\,\text{int}(A_i^C) = 0$. Let $B_i = A_i^C$. Then, $IV\,\text{int}(\bigcup_{i=1}^{\infty} B_i) = 0$, where $IV\,\text{int}(B_i) = 0$. Since (X, τ) is an interval valued vague open hereditarily irresolvable space, $IV\,\text{int}(B_i) = 0$ implies that $IV\,\text{int}(IVcl(B_i)) = 0$. Hence B_i is an interval valued vague nowhere dense set in (X, τ). Hence $IV\,\text{int}(\bigcup_{i=1}^{\infty} B_i) = 0$, where B_i's are interval valued vague nowhere dense set in (X, τ), implies that (X, τ) is an interval valued vague Baire space.

Theorem 3.20: If the interval valued vague topological space (X, τ) is an interval valued vague σ- Baire space and interval valued vague open hereditarily irresolvable, then (X, τ) is an interval valued vague Baire space.

Proof: Let (X, τ) be an interval valued vague σ- Baire space and interval valued vague open hereditarily irresolvable space. Then, $IV\,\text{int}(\bigcup_{i=1}^{\infty} A_i) = 0$ where A_i's are interval valued vague σ-nowhere dense set in (X, τ).By Theorem 3.12, A_i's are interval valued vague dense set in (X, τ). Hence, $IV\,\text{int}(\bigcup_{i=1}^{\infty} A_i) = 0$, where A_i's are interval valued vague dense set in (X, τ). Therefore (X, τ) is an interval valued vague Baire space.

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